KATHOLISCHE UNIVERSITÄT
EICHSTÄTT-INGOLSTADT

Mathematisches Kolloquium

Harald Bohr meets George Boole

Prof. Dr. Andreas Defant

Universität Oldenburg

Mittwoch, 17. Juli 2019, KG I/Bau A 103, 17.00 Uhr s.t.
Um 16.30 Uhr werden im selben Raum Kaffee und Tee serviert.

MATHEMATISCH-GEOGRAPHISCHE FAKULTÄT

Abstract:

Harald Bohr's power series theorem states that for a holomorphic function f on the open unit disc $\mathbb{D}$ we have

$$\sum_{n=0}^{\infty} \frac{|f^{(n)}(0)|}{3^n} \leq \sup_{z \in \mathbb{D}} |f(z)| \,,$$

and here the 3 can not be improved. This nowadays well-known result came out as a sort of by-product of Bohr's intensive study of ordinary Dirichlet series $\sum_n a_n n^{-s}$ from the beginning of the last century, and since then it remained a subject of special attention in various more general settings as e.g. for holomorphic functions on the polydisc $\mathbb{D}^N$. In our talk we intend to report on a recent study of Bohr's phenomenon for real functions on the Boolean cube $\{-1,1\}^N$. Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S \subset \{1,\dots,N\}} \widehat{f}(S) x^S$, where $x^S = \prod_{k \in S} x_k$. Given a class $\mathcal{F}$ of functions on the Boolean cube $\{-1,1\}^N$, the Boolean radius of $\mathcal{F}$ is defined to be the largest $\rho \geq 0$ such that $\sum_S |\widehat{f}(S)| \rho^{|S|} \leq \|f\|_\infty$ for every $f \in \mathcal{F}$. We indicate the precise asymptotic behaviour of the Boolean radius of several natural subclasses, as e.g. the class of all real functions on $\{-1,1\}^N$ or the subclass made of all homogeneous functions. Compared with the classical complex situation subtle differences as well as striking parallels occur, and moreover a curious link to the efficiency of quantum computers occurs.